Depth-first search
COMS20010 (Algorithms II)

John Lapinskas, University of Bristol

Path-finding

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E.g. can an enemy attack the base without breaking down a wall?

Often we want to know the **shortest** path from x to y — see next video!

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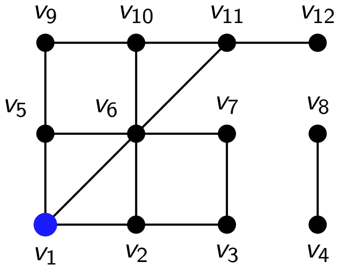

Input: v_1

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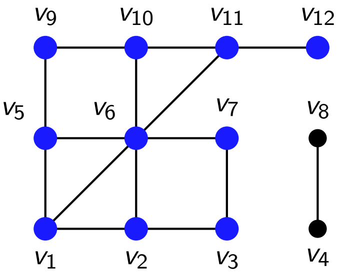

Input: v_1

Output: $[v_1, v_2, v_3, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}]$

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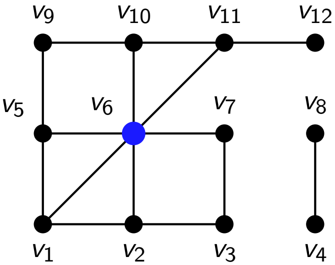

Input: v_6

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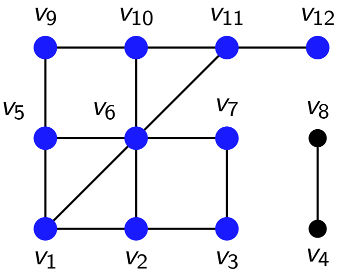

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Output: $[v_1, v_2, v_3, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}]$

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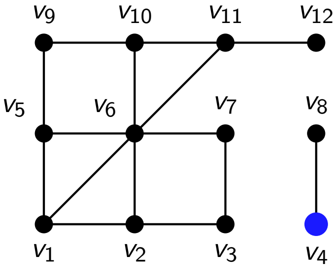

Input: v_4

Component-finding

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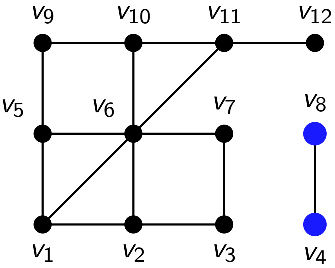

Input: v_4
Output: $[v_4, v_8]$

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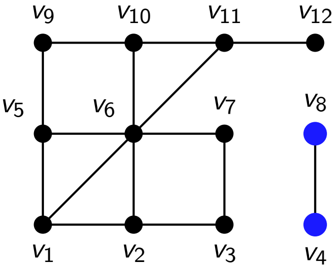

Input: v_4
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In other words, we check whether there is a path from x to y for **all** y.
Turns out the worst-case running time is the same either way!

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Input: A graph G and a vertex $x \in V(G)$.
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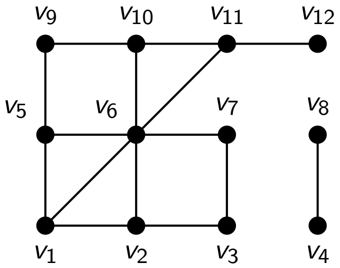
Idea: Think of the graph as like a **maze**: explore greedily until everything looks familiar, then backtrack.



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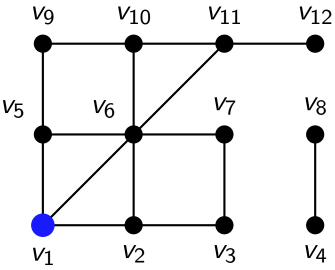

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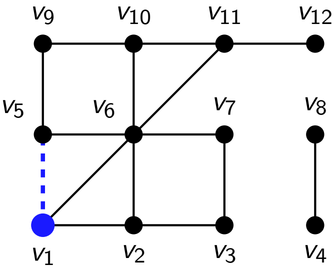

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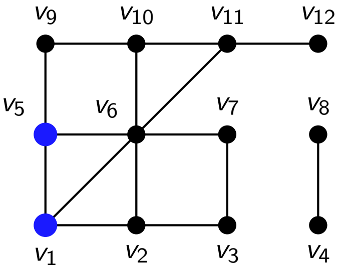

Input: G, v_1
Output: $[v_1, v_5$

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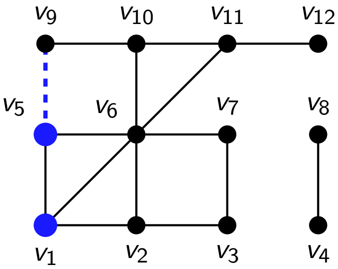

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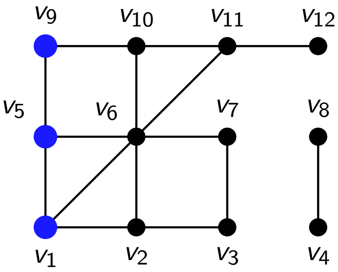

Input: G, v_1
Output: $[v_1, v_5, v_9$

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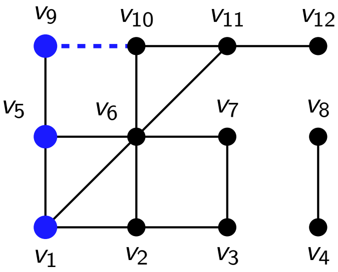

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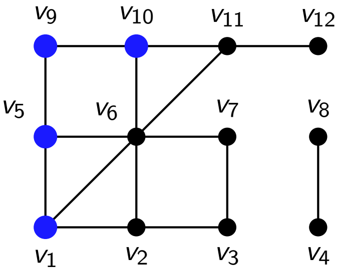

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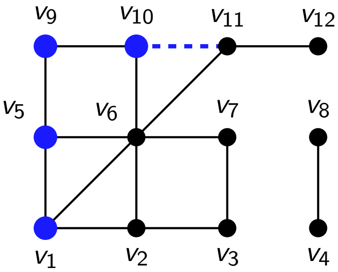

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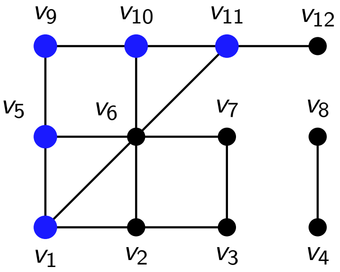

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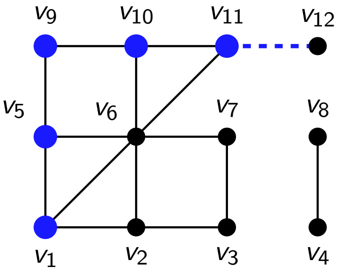

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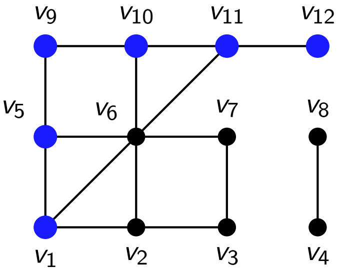

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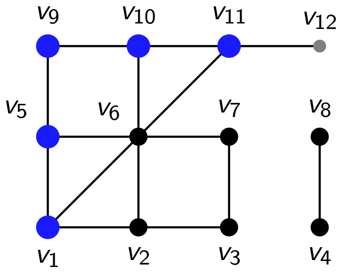

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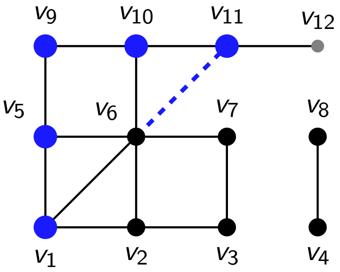

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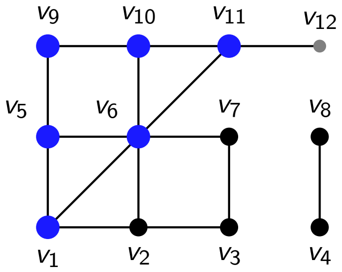

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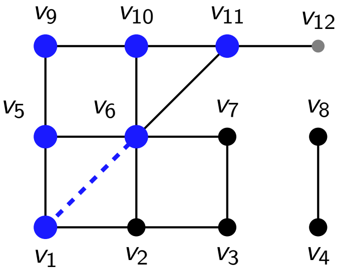

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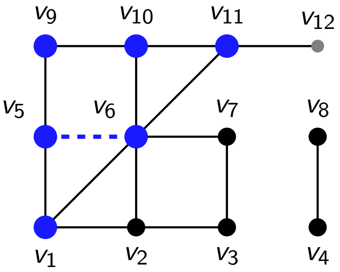

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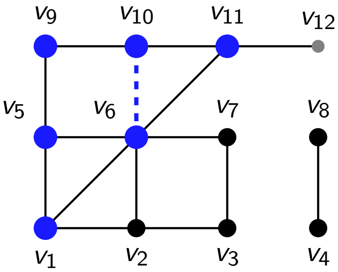

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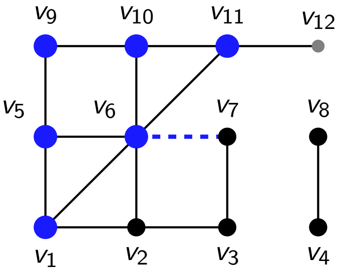

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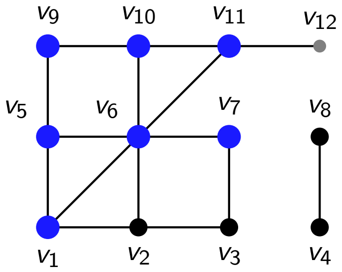

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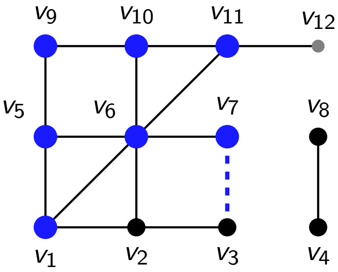

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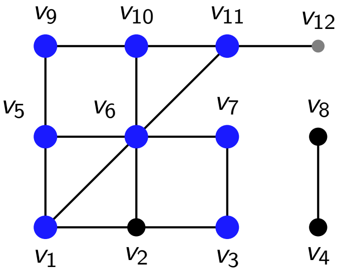

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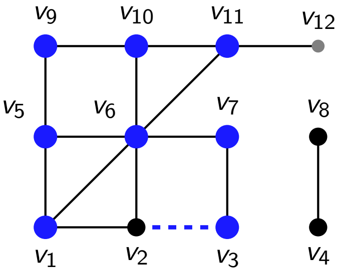

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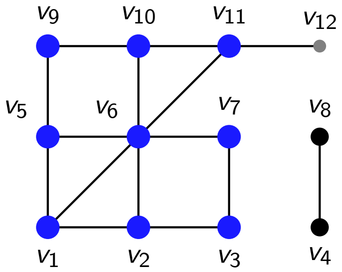

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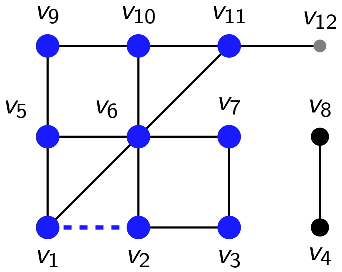

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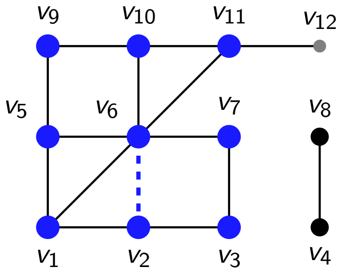

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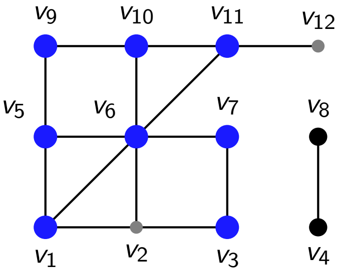

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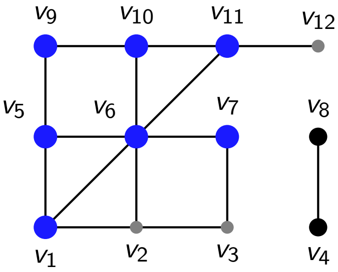

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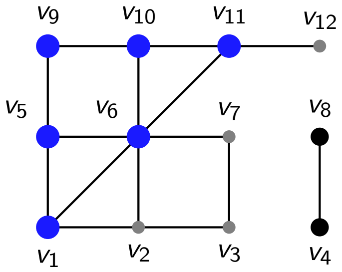

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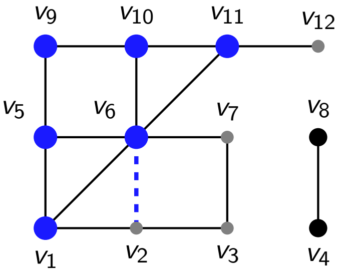

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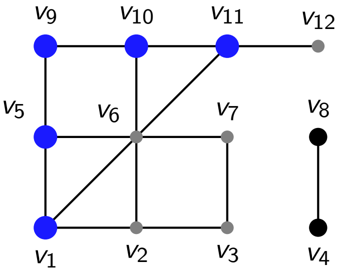

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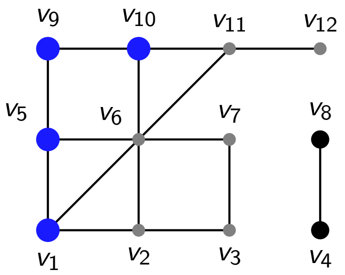
Output: $[v_1, v_5, v_9, v_{10}, v_{11}, v_{12}, v_6, v_7, v_3, v_2$

Depth-first search: The idea

Input: A graph G and a vertex $x \in V(G)$.
Output: A list of all vertices in the component of G containing x.

Idea: Think of the graph as like a **maze**: explore greedily until everything looks familiar, then backtrack.



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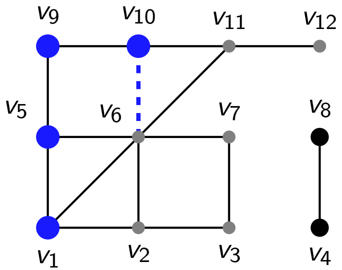

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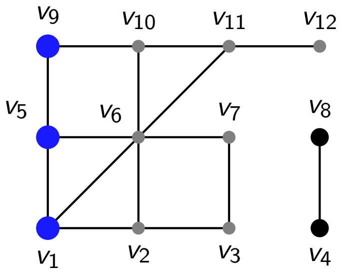

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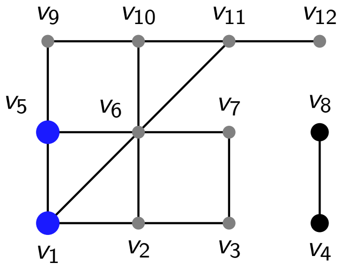

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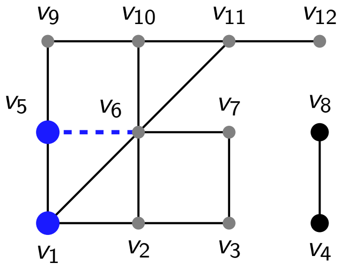

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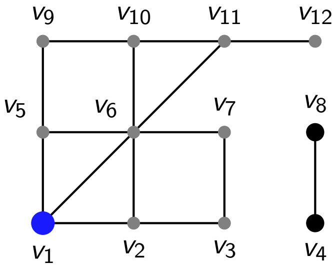

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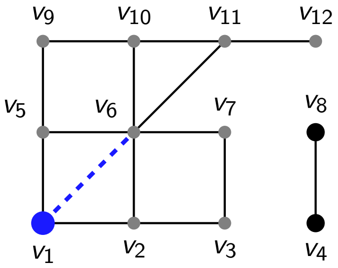

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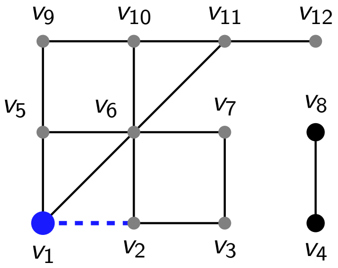

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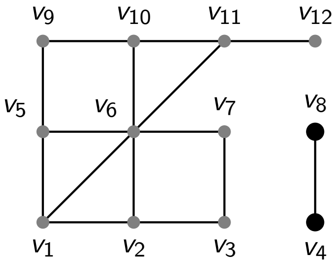

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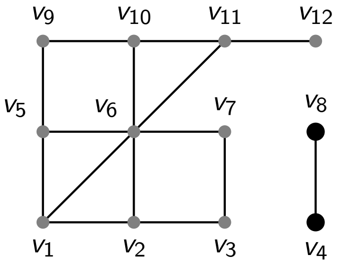

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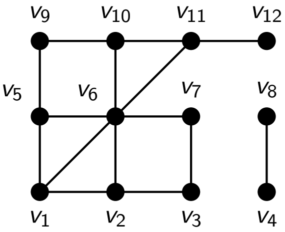
The slick way to implement this is to use recursion.

Pseudocode and example



Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

Number the vertices of G as $v_1, \ldots, v_n$.
Let explored$[i] \leftarrow 0$ for all $i \in [n]$.
Procedure `helper(`v_i`)`
    if explored$[i] = 0$ **then**
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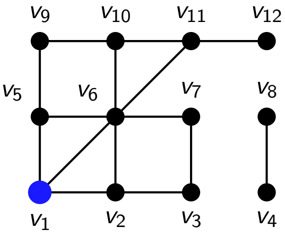
Return $[v_i :$ `explored`$[i] = 1]$ (in some order).

Pseudocode and example



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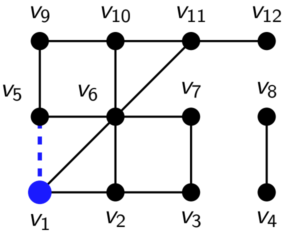

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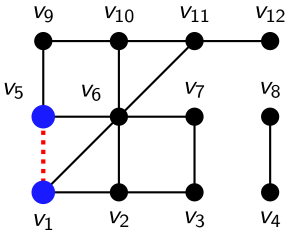

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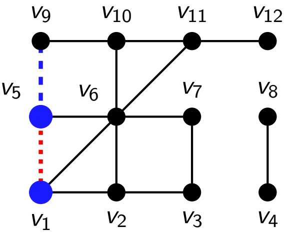

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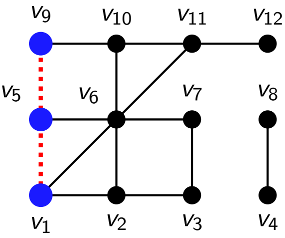

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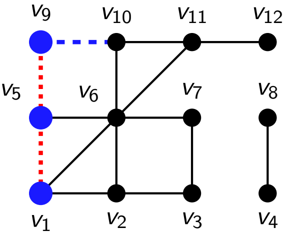

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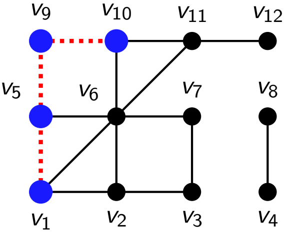

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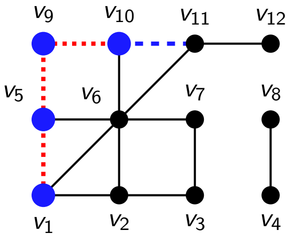

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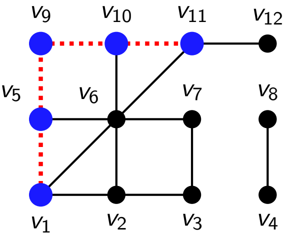

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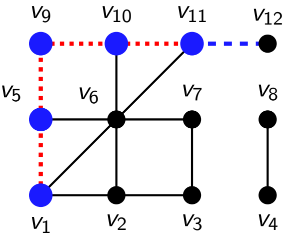

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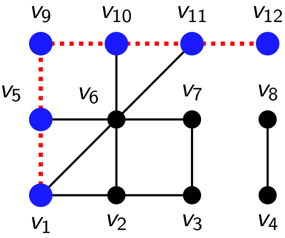

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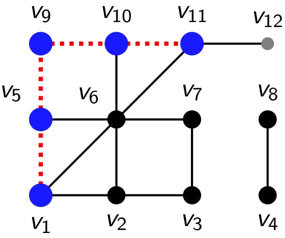

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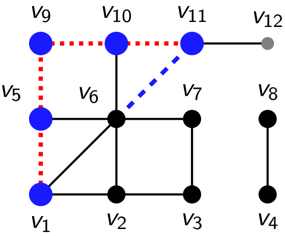

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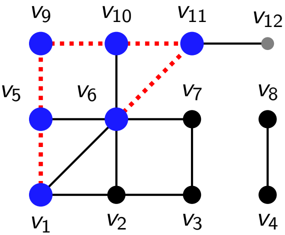

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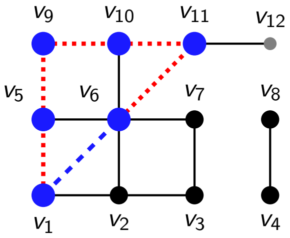

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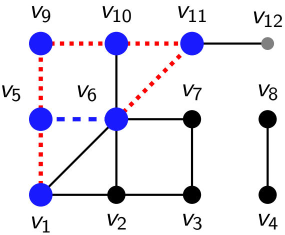

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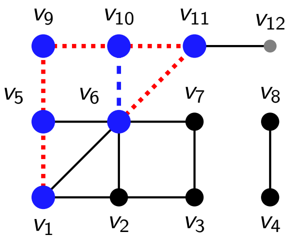

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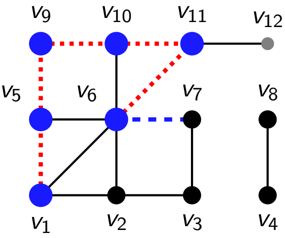

Input: v_1

Algorithm: DFS

Input : Graph $G = (V, E)$, vertex $v \in V$.
Output : List of vertices in v's component.

Number the vertices of G as $v_1, \dots, v_n$.
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Return $[v_i :$ `explored`$[i] = 1]$ (in some order).

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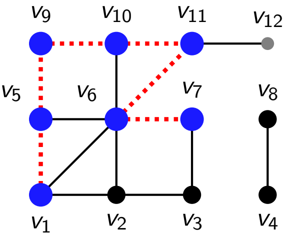

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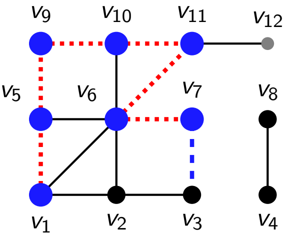

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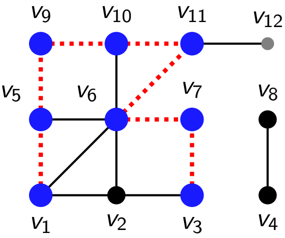

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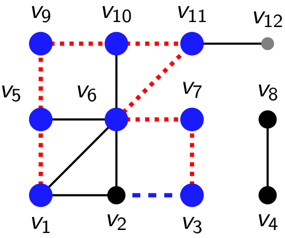

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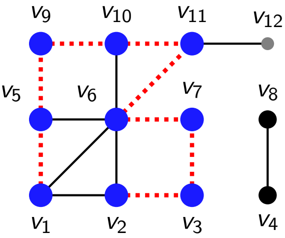

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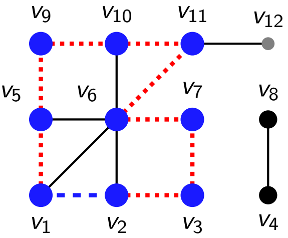

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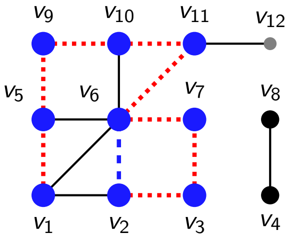

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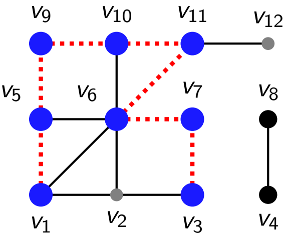

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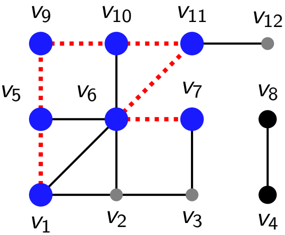

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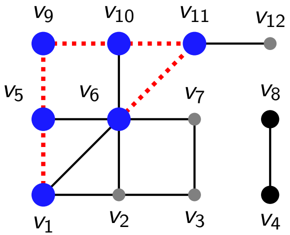

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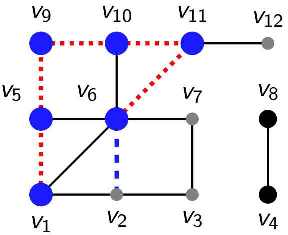

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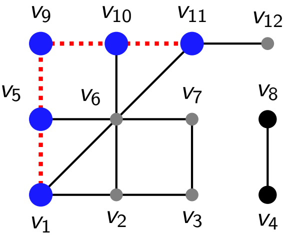

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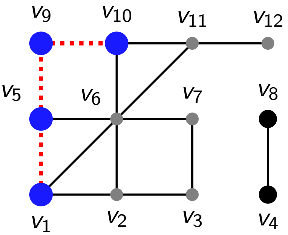

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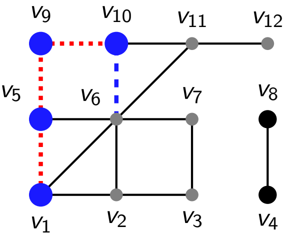

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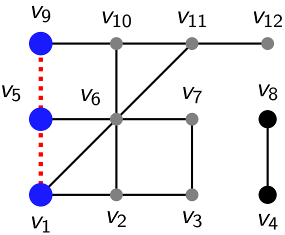

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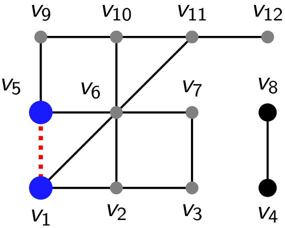

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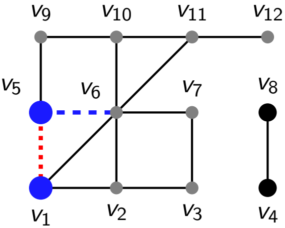

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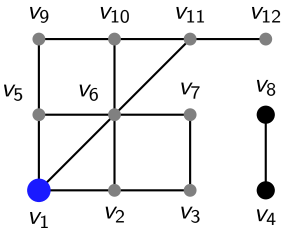

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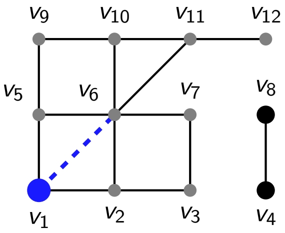

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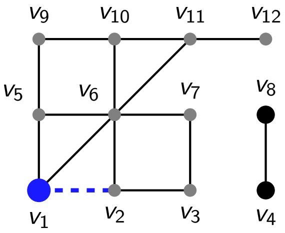

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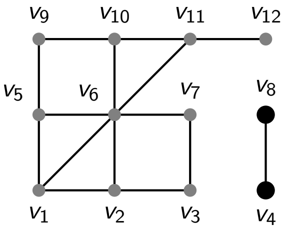

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4      if explored[i] = 0 then
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6          for v_j adjacent to v_i do
7              if explored[j] = 0 then
8                  Call helper(v_j).
9  Call helper(v).
10 Return [v_i : explored[i] = 1] (in some order).
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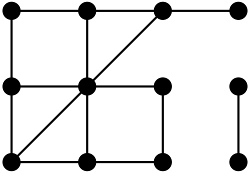
We assume G is in adjacency list form.

Time analysis: In total there are $\sum_{v \in V} d(v) = O(|E|)$ calls to `helper` (each vertex only runs lines 5–7 once), and there is $O(1)$ time between calls. So the running time is $O(|V| + |E|)$.

Correctness I: Output is contained in v's component C



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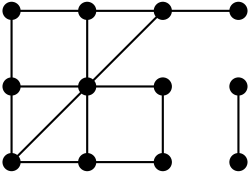

Invariant: "When `helper` is called, if `explored`$[i] = 1$ then $v_i \in V(C)$."

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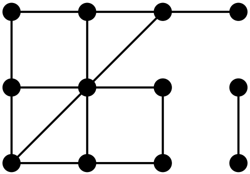

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Proof by induction. Vacuously true for initial call and second call. ✓

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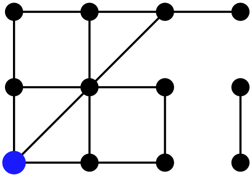

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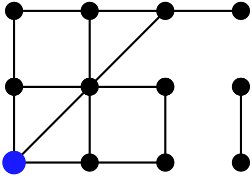

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Suppose it holds at the start of some call `helper`(v_j) from `helper`(v_i).
If v_j is already explored, we're done.

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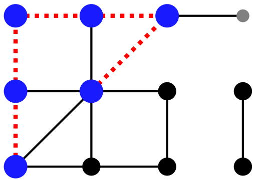

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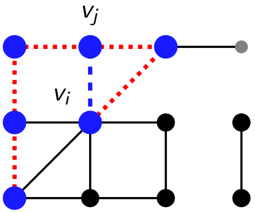

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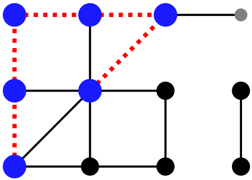

Invariant: "When helper is called, if $\texttt{explored}[i] = 1$ then $v_i \in V(C)$."

Proof by induction. Vacuously true for initial call and second call. ✓

Suppose it holds at the start of some call $\texttt{helper}(v_j)$ from $\texttt{helper}(v_i)$.
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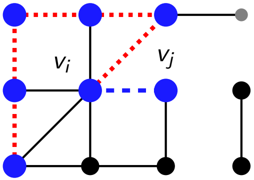

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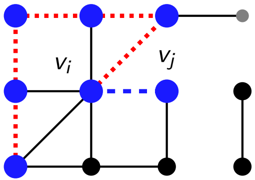

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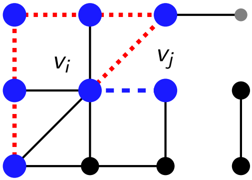

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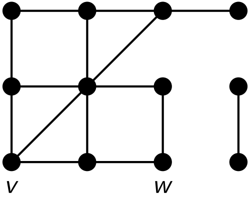
By induction there is a path P from v to v_i. Then Pv_iv_j is a walk from v to v_j, which contains a path, so $v_j \in V(C)$. □

Correctness II: Output contains v's component C



Let $w \in V(C)$. Then there is a path $P = x_1 \dots x_t$ from v to w.

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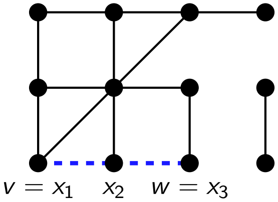

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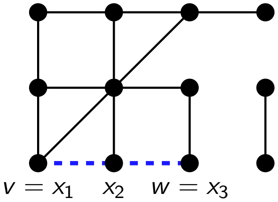

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Claim: Every vertex in P is explored.

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x_1 is explored.

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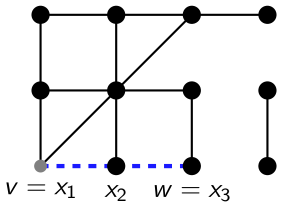

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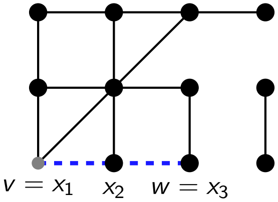

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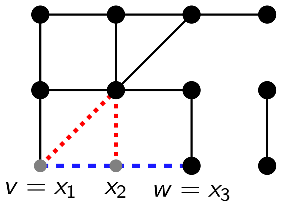

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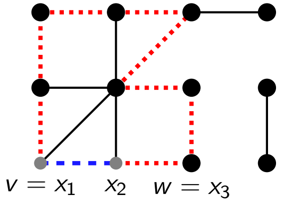

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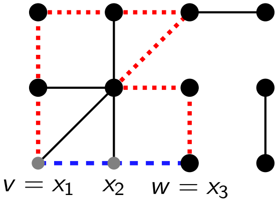

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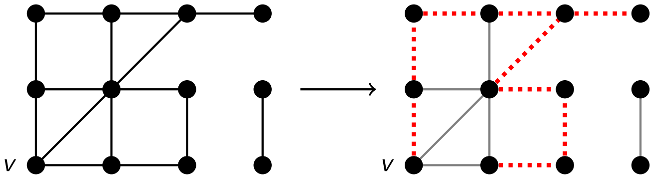
Depth-first search trees

Consider the subgraph formed by the edges **traversed** in DFS:



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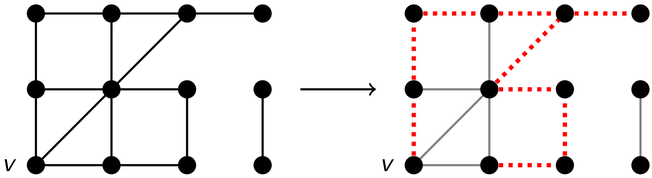

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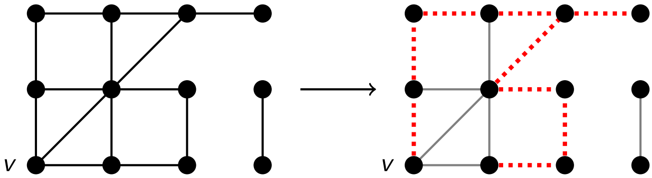

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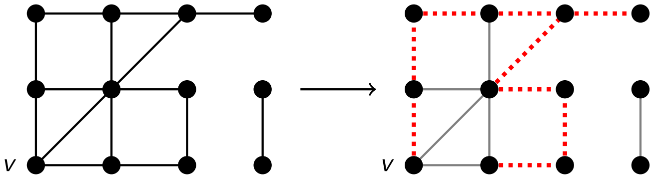

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Depth-first search works for directed graphs too, in exactly the same way.
But paths **between** v and w are replaced by paths **from** v **to** w.